Name:______________________

> This homework is due Thursday, June 8th during recitation. If you have questions regarding any of this, feel free to ask during office hours or send me an email. When writing solutions, present your answers clearly and neatly, showing only necessary work.

1. Find a formula for the Riemann sum obtained by dividing the interval $[a,b]$ into n equal subintervals and using the right-hand endpoint to determine the height of the rectangles. Then take a limit of these sums as $n \to \infty$ to calculate the area under the curve over $[a,b]$.

   (a) $f(x) = 2x$ over the interval $[0,3]$

$$\frac{b-a}{n} = \frac{3-0}{n} = \frac{3}{n} \qquad \sum_{j=1}^{n} \Delta x\, f(a+j\Delta x)$$

$$\sum_{j=1}^{n} 2\left(0+\frac{3}{n}j\right)\frac{3}{n} = \frac{3}{n}\sum_{j=1}^{n}\frac{6}{n}j = \frac{18}{n^2}\sum_{j=1}^{n} j = \frac{18}{n^2}\cdot\frac{n(n+1)}{2}$$

$$= 9\left(\frac{n+1}{n}\right)$$

Answer: $9\left(\frac{n+1}{n}\right)$

$$\lim_{n\to\infty} 9\left(\frac{n+1}{n}\right) = 9$$

Answer: 9

(b) $f(x) = 4x^2$ over the interval $[2, 5]$

$\frac{b-a}{n} = \frac{5-2}{n} = \frac{3}{n}$ $\quad \sum_{j=1}^{n} F(a+j\Delta x)\Delta x$

$\sum_{j=1}^{n} \frac{3}{n} 4\left(2+\frac{3}{n}j\right)^2$

$= \frac{12}{n}\sum_{j=1}^{n} 4 + \frac{12}{n}j + \frac{9}{n^2}j^2 = \frac{12}{n}\sum_{j=1}^{n} 4 + \frac{12}{n}\sum_{j=1}^{n}\frac{12}{n}j + \frac{12}{n}\sum_{j=1}^{n}\frac{9}{n^2}j^2$

$= \frac{48}{n}\sum_{j=1}^{n} 1 + \frac{144}{n^2}\sum_{j=1}^{n} j + \frac{108}{n^3}\sum_{j=1}^{n} j^2$

$= \frac{48}{n}n + \frac{144}{n^2}\frac{n(n+1)}{2} + \frac{108}{n^3}\frac{n(n+1)(2n+1)}{6}$

Answer:

$\lim_{n\to\infty}$ $\quad = 48 + 72 + \frac{108}{3} = 120 + 36$

Answer: 156.

2. Suppose that f and g are integrable and that

$$\int_1^9 f(x)\,dx = -1, \qquad \int_7^9 f(x)\,dx = 5, \qquad \int_7^9 g(x)\,dx = 4.$$

Use the properties of the definite integral to find

(a) $\int_1^9 -2f(x)\,dx$

Answer: 2

(b) $\int_7^9 f(x) + g(x)\,dx$

Answer: 9

(c) $\int_7^9 2f(x) - 3g(x)\,dx$

Answer: -2

(d) $\int_1^7 f(x)\,dx$

Answer: -6

(e) $\int_9^7 -g(x)\,dx$

Answer: 4

3. Show that the value of $\int_0^1 \sqrt{x+8}\,dx$ lies between $2\sqrt{2} \approx 2.8$ and 3.

$$\min(f)(b-a) \le \int_a^b f(x)\,dx \le \max(f)(b-a)$$

$$2\sqrt{2} \cdot 1 \le \int_0^1 \sqrt{x+8}\,dx \le 3 \cdot 1$$

$$2\sqrt{2} \le \int_0^1 \sqrt{x+8}\,dx \le 3$$

4. Evaluate the following integrals.

(a) $\int_{-1}^{1} x^{299}\,dx$

odd

or

$\frac{1}{300}x^{300}\Big|_{-1}^{1} = \frac{1}{300}(1-1) = 0$

Answer: 0

(b) $\int_{-2}^{3} x^3 - 2x + 3\,dx$

$\frac{1}{4}x^4 - x^2 + 3x\Big|_{-2}^{3} = \frac{81}{4} - 9 + 9 - \left(\frac{16}{4} - 4 - 6\right)$

$= \frac{65}{4} + 10$

Answer: $105/4$

5. Evaluate the following integrals.

(a) $\int_0^{\pi} 1 + \cos(x)\,dx$

$x + \sin(x)\Big|_0^{\pi} = \pi + 0 - 0 - 0$

Answer: π

(b) $\int_0^{\pi/3} 4\frac{\sin(x)}{\cos^2(x)}\,dx$

$\Longrightarrow -4\int \frac{1}{u^2}\,du = \frac{4}{u}\Big|_{x=0}^{x=\pi/3}$

$u = \cos(x)$

$du = -\sin(x)dx$

$-du = \sin(x)dx$

$= \frac{4}{\cos(x)}\Big|_0^{\pi/3} = \frac{4}{\cos(\pi/3)} - \frac{4}{\cos(0)}$

$= 4/\tfrac{1}{2} - 4 = 8 - 4$

Answer: $\boxed{4}$